

# METHOD OF A REGULAR REGIME FOR THE DETERMINATION OF VARIABLE THERMAL COEFFICIENTS

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**Abstract**—The present work deals with the application of the theory and methods of a regular thermal regime in the case of cooling (heating) a body at variable thermal coefficients. It is shown that the description of a process by the regular regime formulae is possible and makes sense but in this case thermal coefficients determining the rate  $m$  are averaged in some temperature interval  $\bar{\Delta}t$ . The value  $\bar{\Delta}t$  increases with  $Bi$  and at mean  $Bi$  it approaches the value of the initial excessive temperature  $\vartheta_0$ .

Therefore, the regular regime method is suitable for the determination of the variable heat capacity  $c_p(t)$  and the variable heat transfer coefficient  $\alpha(\vartheta)$  at low  $Bi$ , but makes no sense for the determination of the variable thermal conductivity  $\lambda(t)$  at mean and high  $Bi$ .

In the present work the method of variable coefficients is applied to investigating heat capacity of metals in a region of phase conversions of the second kind and heat transfer of a spherical specimen in various media and at various temperatures.

**Résumé**—Le travail actuel concerne l'application de la théorie et des méthodes du régime thermique régulier dans le cas du refroidissement (ou du chauffage) d'un corps à propriétés thermiques variables. On montre qu'il est possible, grâce aux formules du régime régulier, d'obtenir une représentation valable dans le cas où les coefficients thermiques déterminant la vitesse  $m$  sont pris en moyenne dans un intervalle de température  $\bar{\Delta}t$ . La valeur de  $\bar{\Delta}t$  augmente avec  $Bi$  et, comme  $Bi$ , approche de la valeur de l'écart de température initial  $\vartheta_0$ .

Par suite, la méthode du régime régulier peut s'employer pour la détermination d'une chaleur spécifique variable  $c_p(t)$  et d'un coefficient de transmission de chaleur variable  $\alpha(\vartheta)$  pour de faibles valeurs de  $Bi$ , mais n'a pas de signification pour la détermination d'une conductibilité thermique variable  $\lambda(t)$ .

Dans cette étude, la méthode des coefficients variables est appliquée à la détermination de la chaleur spécifique des métaux au voisinage des conversions de phase de 2ème espèce, à la détermination du transfert de chaleur d'un modèle sphérique de milieux différents à différentes températures.

**Zusammenfassung**—Die Arbeit behandelt die Anwendung der Theorie und der Methode des regulären thermischen Zustandes auf die Kühlung (Beheizung) eines Körpers bei veränderlichen Wärmeoeffizienten. Es wird gezeigt, dass die Beschreibung eines Vorganges durch die Formeln dieser Theorie möglich und dann sinnvoll ist, wenn die Koeffizienten, die das Verhältnis  $m$  bestimmen, über ein Temperaturintervall  $\bar{\Delta}t$  gemittelt werden. Der Wert  $\bar{\Delta}t$  nimmt mit wachsendem  $Bi$  zu und erreicht bei einem mittleren  $Bi$  den Wert der anfänglichen Übertemperatur  $\vartheta_0$ .

Die Methode des regulären thermischen Zustandes ist zur Bestimmung veränderlicher spezifischer Wärmen  $c_p(t)$  und veränderlicher Wärmeübergangskoeffizienten  $\alpha(\vartheta)$  bei kleinen  $Bi$  anwendbar, sie ist jedoch zur Bestimmung veränderlicher Wärmeleitahlen  $\lambda(t)$  unbrauchbar.

Die Methode der veränderlichen Koeffizienten wird angewandt auf die Untersuchung der spezifischen Wärmen von Metallen im Bereich der Phasenumwandlung zweiter Art und auf den Wärmeübergang von einem kugelförmigen Probekörper in unterschiedlichen Medien bei verschiedenen Temperaturen.

**Аннотация**—Данная работа посвящена применению теории и методов регулярного теплового режима на случай охлаждения (нагрева) тела при переменных тепловых коэффициентах. Показано, что описание процесса формулами регулярного режима возможно и имеет смысл, но при это и тепловые коэффициенты, определяющие темп  $m$ , усредняются в некотором температурном интервале  $\bar{\Delta}t$ . Величина  $\bar{\Delta}t$  возрастает вместе с  $Bi$  и уже при средних значениях  $Bi$  приближается к величине начальной избыточной температуры  $\vartheta_0$ .

Поэтому метод регулярного режима пригоден для определения переменной теплоемкости  $c_p(t)$  и переменного коэффициента теплоотдачи  $\alpha(\vartheta)$  при малых  $Bi$ , но не имеет смысла для определения переменной теплопроводности  $\lambda(t)$  при средних и больших  $Bi$ .

В настоящей работе предложенный метод переменных коэффициентов применяется для исследования теплоемкости металлов в области фазовых превращений второго рода и теплообмена сферического образца в различных средах и при разных температурах.

#### NOMENCLATURE

$t$ ,	temperature;
$\tau$ ,	time;
$\vartheta$ ,	excessive temperature;
$t_f$ ,	temperature of medium;
$t_w$ ,	temperature of surface;
$a$ ,	thermal diffusivity;
$c_p$ ,	heat capacity;
$\lambda$ ,	thermal conductivity;
$\alpha$ ,	heat transfer coefficient;
$m$ ,	rate of cooling (heating);
$Bi$ ,	Bio criterion;
$R$ ,	radius of specimen;
$f(r)$ ,	initial temperature distribution;
$P_n$ ,	roots of the characteristic equation;
$C_n$ ,	coefficients of expansion;
$A_n = C_n \frac{P_n}{R}$ ,	thermal amplitudes;
$\Delta t$ ,	temperature interval of averaging of thermal coefficients;
$\Delta t_{\text{reg}} (\tau_{\text{reg}})$ ,	interval of regularization;
$\Delta t_{ik} (\tau_{ik})$ ,	interval of treating the curve $\vartheta(\tau)$ ;
$c_{p, \text{max}}$ ,	height of the heat capacity peak;
$\Delta c_{p, \text{max}}$ ,	heat capacity jump at the Curie point;
$\Delta t_R$ ,	spatial temperature drop.

#### 1. INTRODUCTION

THE theory and methods of a regular thermal regime of the first kind, developed by Kondratiev [1] and his disciples, are widely applied at present to the investigation of heat transfer processes and to the determination of thermal properties of various materials. The design formulae of the regular regime methods were obtained on the assumption that coefficients of thermal diffusivity  $a$ , thermal conductivity  $\lambda$  and of heat transfer  $\alpha$  entering both the Fourier equation and a boundary condition of the third

kind are constant. It is of interest to extend the regular regime method to the determination of variable thermal coefficients. Lately, much attention has been given to this problem [2-7].

The idea of the method lies in the fact that the curve of cooling (heating) a specimen  $\vartheta = f(\tau)$  in the medium with constant temperature is divided into separate intervals  $\Delta\tau_{ik}$ ,  $\Delta t_{ik}$  in which the rate

$$m_{ik} = \frac{\ln \vartheta_i / \vartheta_k}{\Delta\tau_{ik}} \quad (1)$$

is determined. In each interval the thermal coefficients  $a$ ,  $\lambda$ ,  $c$ ,  $\alpha$  and the rate  $m_{ik}$  are assumed to be constant values, and calculations are carried out by general formulae of the regular regime. Thus, having determined  $m = f(\vartheta)$  for the investigated specimen and  $\alpha = f(\vartheta)$  for the given medium, we can obtain the dependences  $c_p = f(\vartheta)$ ,  $a$ ,  $\lambda = f(\vartheta)$  sought for or, vice versa, if we know  $a$ ,  $\lambda = f(\vartheta)$  for the specimen we can determine  $\alpha = f(\vartheta)$ .

Experimental works [2, 5] consider some particular cases of the determination of variable coefficients and, in the main, of heat transfer coefficient. There are attempts to apply this method to the determination of the variable heat conduction coefficient [5]. However, we cannot find a sufficient analysis or substantiation of the method in the mentioned works.

In theoretical works [6, 7] integral parameters which make it possible to obtain the expression of regularity in a general case of variable thermophysical coefficients were substituted into the heat conduction equation instead of temperature and time. However, this transformation does not yet allow one to propose a concrete method of an experimental determination of variable coefficients (with the exception of a very bulky method of determining  $\alpha$ ), and the method of dividing the curve  $\vartheta(\tau)$  into separate intervals with the application of the regular

regime equations in a general form preserves its importance.

The present work considers and analyses this method in its general form. The experimental data which can be of original interest are presented here.

2. TEMPERATURE INTERVAL OF AVERAGING OF THERMAL COEFFICIENTS

(1) When substantiating the method it is necessary to ascertain at what actual temperature interval  $\bar{\Delta t}$  the thermal coefficients which determine the rate  $m_{ik}$  are averaged. This interval of averaging the thermal coefficients  $\bar{\Delta t}$  must differ in size from  $\Delta t_{ik}$ . If we do not take into account this fact, dividing into intervals may appear to be a formal procedure without its physical sense. The value  $\Delta t$  should serve as the most important characteristic of the regular regime method for the variable coefficients, since it points out the expediency of the application of this method in various regimes.

The interval of averaging the thermal constants  $\lambda$ ,  $c_p$  and  $\alpha$  of the specimen is determined by a temperature drop  $\Delta t_R$  along the cross-section of the specimen, by regime regularization time  $\tau_{reg}$  (with the corresponding interval  $\Delta t_{reg}$ ) and by the value of the interval of treating the curve  $\Delta t_{ik}(\Delta \tau_{ik})$ .

In fact, if the time of the regularization of the thermal regime has a finite value, then the rate  $m_{ik}$  measured at the interval  $\Delta t_{ik}$  reflects values of the thermal coefficients in the preceding time interval  $\tau_{reg}$  and in the corresponding temperature interval  $\Delta t_{reg}$ . At the same time, while measuring the rate  $m_{ik}$  in the specimen there is a spatial temperature drop  $\Delta t_R$  which is also one of the factors determining the value  $\bar{\Delta t}$ .

Figure 1 demonstrates graphically a value  $\bar{\Delta t}$  of the total interval of averaging at the temperature measurement in the centre of the specimen being cooled:

$$\bar{\Delta t} = \begin{cases} \Delta t_{reg} + \frac{1}{2}\Delta t_{ik} + [\Delta t_R]_k & \text{at } \Delta t_{reg} > \frac{1}{2}\Delta t_{ik} & (2a) \\ \Delta t_{ik} + [\Delta t_R]_k & \text{at } \Delta t_{reg} \leq \frac{1}{2}\Delta t_{ik} & (2b) \end{cases}$$

Thus, the temperature interval of averaging thermal constants depends only partially on the

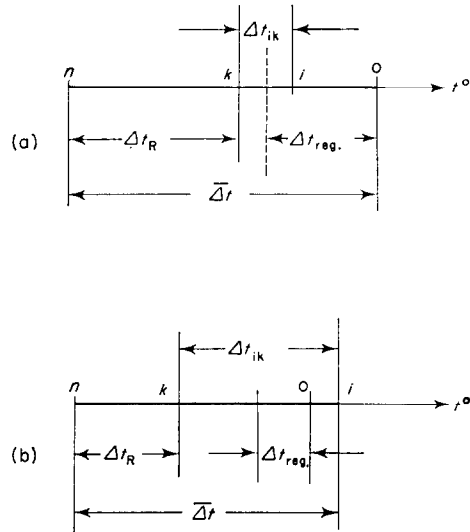


FIG. 1. Temperature interval of averaging of thermal constants  $\bar{\Delta t}^\circ$ .

way of the treatment of experimental curves  $\vartheta(\tau)$  (i.e. on the value  $\Delta t_{ik}$ ). The thermal constants are being averaged even at the differentiation of curves ( $\Delta t_{ik} \rightarrow 0$ ) in the required minimum interval

$$\bar{\Delta t}_{min} = \Delta t_{reg} + [\Delta t_R]_k. \tag{3}$$

Let us consider in detail each term determining the value  $\bar{\Delta t}_{min}$ . All the calculations will be made for specimens of a spherical form. The conclusions can be extended to a body of any form.

(2) Regularization interval

As is known, the general solution of the Fourier equation has the form

$$\vartheta(r, \tau) = \sum_{n=1}^{\infty} C_n U_n \exp(-m_n \tau) \tag{4}$$

where

$$U_n = \frac{\sin p_n(r/R)}{r}$$

are eigenfunctions for a sphere ( $U_n R/p_n = 1$  at the temperature measurement in the centre of the

sphere),  $p_n$  are the roots of the characteristic equation

$$Bi = \frac{aR}{\lambda} = 1 - pctgp, \quad (5)$$

the numbers

$$m_n = \frac{p_n^2 a}{R^2}, \quad (6)$$

$$C_n = \frac{\int_0^R r f(r) \sin p_n(r/R) dr}{\int_0^R \sin^2 p_n(r/R) dr} \quad (7)$$

are the constant coefficients of expansion, and the integrand  $f(r) = \vartheta(r)|_{\tau=0}$  describes an initial temperature distribution in the specimen.

The regular thermal regime comes after the expiry of the time  $\tau_{\text{reg}}$  of the process regularization when it is possible to neglect all the terms of (4) except the first one:

$$|C_1 U_1 \exp(-m_1 \tau)| \gg \left| \sum_{n=2}^{\infty} C_n U_n \exp(-m_n \tau) \right|.$$

The theoretical evaluation of the time  $\tau_{\text{reg}}$  or of the continuity of an irregular regime has not been carried out up to now and was thought to be unrealized [1]. However, the value  $\tau_{\text{reg}}$  and the corresponding temperature regularization interval may be calculated if we take some finite quantity of the ratio

$$|\Delta|_{\text{per cent}} = \left| \frac{\sum_{n=2}^{\infty} C_n p_n \exp(-m_n \tau_{\text{reg}})}{C_1 p_1 \exp(-m_1 \tau_{\text{reg}})} \right| \cdot 100. \quad (8)$$

It is reasonable to take  $|\Delta| = 4$  per cent (since the usual experimental error is  $\Delta m/m = \pm 2$  per cent).

The value  $\tau_{\text{reg}}$  is calculated by equation (8). The numbers  $p_n$ ,  $m_n$  are determined by (5), the coefficients  $C_n$  are calculated by formula (7).

Two main and practically important cases should be considered:

$$(a) \quad f(r) = \vartheta_0 = \text{const} \quad (9)$$

a body is heated uniformly at the initial moment; the period of the regime regularization at the beginning of a process is to be found.

$$(b) \quad f(r) = C'_1 \frac{\sin p'_1(r/R)}{r} \quad (10)$$

at the initial moment in the specimen there is a regular regime established in the previous interval  $\Delta t'_{ik}$  and corresponding to some values of the thermal coefficients  $a'$ ,  $\lambda'$ ,  $c'$ ,  $a'$  and of the criterion  $Bi'$ . A period of the regime regularization at the transition into a new interval  $\Delta t_{ik}$  with the thermal coefficients  $a$ ,  $\lambda$ ,  $c$ ,  $a$  and with the criterion  $Bi$  is to be obtained.

This case\* corresponds to conditions of the method of the variable coefficients.

(3) *The initial condition*  $f(r) = \vartheta_0 = \text{const}$  (9)

For this case the analysis of the value  $\tau_{\text{reg}}$  depending on various factors is made in Table 1. The time of regularization for typical specific regimes is calculated by means of successive approximations with regard for three terms of a series. The values of thermal amplitudes  $A_n = C_n(p_n/R)$  and  $p_n = f(Bi)$  are taken from Tables of [8]. The radius of the specimen is taken equal to  $R = 2$  cm when calculating the numbers  $m_n$  (6). The number  $m_1$  for metallic specimens is in satisfactory agreement with the experimental value of the regular regime rate [4, 9].

The temperature interval of the regularization  $\Delta t_{\text{reg}}$  is given by the formula

$$\Delta t_{\text{reg}} = \vartheta_0 [1 - 0.96 A_1 \exp(-m_1 \tau_{\text{reg}})], \quad (11)$$

since

$$\Delta t_{\text{reg}} = \vartheta_0 - \vartheta|_{r=0, \tau=\tau_{\text{reg}}},$$

and

$$\vartheta|_{r=0, \tau=\tau_{\text{reg}}} = \vartheta [A_1 \exp(-m_1 \tau_{\text{reg}}) + \sum_{n=2}^{\infty} A_n \exp(-m_n \tau_{\text{reg}})]. \quad (12)$$

On the basis of Table 1 and (11) we can arrive at some general conclusions about the time of regularization and the value of the interval  $\Delta t_{\text{reg}}$ . The interval  $\Delta t_{\text{reg}}$  increases with  $Bi$  at  $Bi = \text{var}$ ,  $a = \text{idem}$  (for example, when the heat transfer coefficient varies). At  $a = \text{var}$ ,  $Bi = \text{idem}$  (for example, when heat capacity of the specimen changes in the process of phase conversion) the roots of characteristic equation (5)  $p_n$  and the

\* G. N. Dulnev draws our attention to the necessity of its detailed consideration.

Table 1. The determination of the regularization time  $\tau_{\text{reg}}$   
(Spherical specimen  $R = 2$  cm)

	Examples	$a$ (cm <sup>2</sup> /s)	$Bi$	$m_1$ (1/s)	$m_2$ (1/s)	$m_3$ (1/s)	$A_1$	$A_2$	$A_3$	$\tau_{\text{reg}}$ (s)	$\Delta$ per cent
1	1. Metals Carbon steel at the determination of heat capacity near the point $A_0$	0.11	0.2	0.016	0.57	1.65	1.06	-0.09	0.05	1	-3.8
2	Regime of the determination of $a, \lambda$	0.11	4	0.168	0.75	1.85	1.72	-1.23	0.88	5	-3.8
3	The same	0.11	10	0.221	0.89	2.06	1.92	-1.74	1.51	4.7	-3.9
4	Metallic $a$ -calorimeter (practically unrealized case)	0.11	$\infty$	0.271	1.08	2.44	+2.00	-2.00	+2.00	4	-3.8
5	Carbon steel 45, $t_f = 815^\circ\text{C}$ , lead melt is a medium	0.04	4	0.060	0.27	0.67	1.72	-1.23	0.88	14	-3.8
6	Steel higher the point $A_1$ , $t_f = 670^\circ\text{C}$ , lead melt is a medium	0.05	7	0.090	0.38	0.90	1.87	-1.57	1.28	10.5	-4.1
7	Steel 45 at the increase of the heat capacity peak near the point $A_1$ , $t_f = 670^\circ\text{C}$ , lead melt is a medium	0.025	7	0.045	0.19	0.45	1.87	-1.57	1.28	21	-4.1
8	2. Insulators Microcalorimeter	0.004	0.1	$0.30 \times 10^{-3}$	0.020	0.060	1.03	-0.04	0.03	$\leq 1$	-4
9	$\lambda$ -calorimeter	0.004	4	$6 \times 10^{-3}$	0.027	0.067	1.72	-1.23	0.88	140	-3.8
10	The same	0.004	10	$8 \times 10^{-3}$	0.033	0.075	1.92	-1.71	1.51	130	-4
11	$a$ -calorimeter	0.004	$\infty$	$9.90 \times 10^{-3}$	0.039	0.089	+2.00	-2.00	+2.00	110	-4.1

coefficients  $C_n$  remain invariable and identity (8) leads to the condition  $m_n \tau_{\text{reg}} = \text{idem}$  or  $a \tau_{\text{reg}} = \text{idem}$ . Though, in this case, the time of regularization is inversely proportional to the thermal diffusivity  $a$ , the value of the interval  $\Delta t_{\text{reg}}$  (11) remains constant.

After the establishment of the regular regime in a spherical specimen spatial temperature drop along the radius of it is expressed by the formula

$$\Delta t_R = \vartheta|_{r=0, \tau=\tau_{\text{reg}}} \left( 1 - \frac{\sin p_1}{p_1} \right) \quad (13)$$

where  $\vartheta|_{r=0, \tau=\tau_{\text{reg}}}$  is determined by (12).

Naturally, at  $Bi = \text{var}$ ,  $a = \text{idem}$  the spatial drop increases with  $Bi$ . At  $Bi = \text{idem}$  the values  $A_1, p_1, m_1 \tau_{\text{reg}} = \text{idem}$ , i.e. the drop  $\Delta t_R$  does not change. It means, for example, that if the specimen is under the regular regime conditions, then

the temperature drop along a cross-section of the specimen does not change when the phase conversion of the second kind is going on in it (without heat liberation,  $c = \text{var}$ ,  $\lambda = \text{const}$ ).

On the basis of (11–13) it is possible to write the expression for the minimum temperature interval of averaging thermal constants:

$$\overline{\Delta T}_{\min} = \vartheta_0 \left( 1 - 0.96 A_1 \exp(-m_1 \tau_{\text{reg}}) \cdot \frac{\sin p_1}{p_1} \right). \quad (14)$$

The consideration of formula (14) confirms the conclusions derived when analysing separately  $\Delta t_{\text{reg}}$  and  $\Delta t_R$ : the interval of averaging  $\overline{\Delta T}_{\min}$  increases with the increase in  $Bi$  and does not change with  $c$  and  $a$  (at  $Bi = \text{idem}$ ).

In Table 2 the values of the intervals  $\Delta t_{\text{reg}}$ ,  $\Delta t_R$  and  $\overline{\Delta T}_{\min}$  are calculated depending on the value of  $Bi$ .

Table 2. The values  $\Delta t_{\text{reg}}$ ,  $\Delta t_R$  and  $\overline{\Delta T}_{\min}$  at various  $Bi$

$Bi$	$0.96 A_1 \exp(-m_1 \tau_{\text{reg}})$	$\frac{\sin p_1}{p_1}$	$\frac{\Delta t_{\text{reg}}}{\vartheta_0}$	$\frac{\Delta t_R}{\vartheta_0}$	$\frac{\overline{\Delta T}_{\min}}{\vartheta_0}$
0.2	0.998	0.91	0.002	0.09	0.09
4	0.72	0.26	0.27	0.54	0.81
7	0.68	0.15	0.32	0.58	0.90
$\infty$	0.65	0	0.35	0.65	1.00

It follows from this table that the thermal constants are being averaged almost in the whole temperature interval of cooling (heating) the specimen irrespective of its initial temperature  $\vartheta_0$  already at the mean values of  $Bi = 4 + 7$ , i.e. in the region of determining heat conduction of metals and insulators.

Thus, the method of the variable coefficients is valid for the investigation of the temperature dependence of the body heat capacity  $c_p(t)$  in regimes at low values of  $Bi$ , but loses its sense when one tries to determine the variable heat conductivity  $\lambda(t)$  in regimes at mean and high  $Bi$ .

It is necessary to bear in mind that just the determination of the variable heat capacity is of the greatest interest since  $c_p(t)$  is highly susceptible to structural changes in the specimen

whereas heat conduction varies relatively in low degree and monotonously with the temperature.\*

The temperature interval of averaging the heat transfer coefficient is less than that of the thermal constants since at  $t_f = \text{const}$  the coefficient  $\alpha$  is the function of the body surface temperature ( $\vartheta_w$ ) or of its derivatives, for example, of the average boundary-layer temperature,  $t_m$ .

However, in the regimes at mean and high  $Bi$  the value

$$\vartheta_w = \vartheta|_{r=0, \tau_{\text{reg}}} \frac{\sin p_1}{p_1}$$

is so small at the end of the regularization interval that any attempt at determining  $\alpha = f(\vartheta_w)$  will be senseless. Thus, the determination of the quantitative dependence  $\alpha(\vartheta_w)$  is possible only at low  $Bi$  while at mean and high  $Bi$  it makes no sense as well as the determination of the variable heat conduction.

The analysis of such an important parameter as an interval of averaging the thermal coefficients  $\overline{\Delta T}$ , makes it possible to explain such facts as formal observance of the regular regime at intensive cooling of steel specimens in water and oil in the range of  $\sim 700\text{--}20^\circ\text{C}$  [1], at cooling of refractory materials from  $\sim 1000^\circ\text{C}$  to room temperature in the regimes of  $\lambda$  and  $a$ -calorimeters (in the regimes of calorimeter work on the determination of thermal conductivity and thermal diffusivity).

$$(4) \text{ The initial condition } f(r) = C_1' \frac{\sin p_1'(r/R)}{r}. \quad (10)$$

To obtain the main conclusions on the application of the method of the variable coefficients it was sufficient to consider the temperature interval of averaging at cooling (heating) a body with uniform temperature distribution (Section 3). However, to specify the values  $\tau_{\text{reg}}$  and  $\Delta t_{\text{reg}}$  in the case when thermal coefficients vary in the regular regime already established it is necessary to calculate the expansion coefficients  $C_n$  (7) at the initial condition (10). As was shown

\* The method of determining the variable heat capacity under the conditions of the regular regime is to a certain extent analogous to methods of thermic analysis which are also bound up with the receipt of curves of cooling (heating) an investigated specimen in a wide temperature interval [10–12].

above this case has practical value only for regimes at low  $Bi$ .

Taking into account that

$$C'_1 = \frac{\vartheta_0|_{r=0} \cdot R}{p'_1}$$

(where  $\vartheta_0|_{r=0} = f(0)$  is the temperature of the specimen centre at the beginning of the given interval  $\Delta t_{ik}$ ) we get for  $C_n$ :

$$C_n = \frac{\vartheta_0|_{r=0} \cdot R}{p'_1} \cdot \frac{\int_0^R \sin p'_1(r/R) \sin p_n(r/R) dr}{\int_0^R \sin 2 p_n(r/R) dr} \quad (15)$$

If the change of  $Bi = \alpha R/\lambda$  during a process can be neglected and only heat capacity  $c_p(t)$  of the specimen changes essentially, then  $p'_1 = p_1$  and according to (15)  $C_1 = C'_1$ ;  $C_n = 0$  at  $n = 2, 3, \dots$ . Thus, at  $Bi = idem$ ,  $\alpha = var$  the regularity of the regime is not disturbed, and  $\tau_{reg}, \Delta t_{reg} = 0$  despite the fact that the rate  $m_{ik}(t)$  varies continuously.

However, in real conditions the value  $Bi$  usually changes somewhat, mainly because of the change of the heat transfer coefficient  $\alpha$ . Then the values  $p'_1, p_n$  are the roots of two different equations of the type (5), integrands  $\sin p'_1(r/R), \sin p_n(r/R)$  do not form an orthogonal set, and, therefore, all the coefficients  $C_n \neq 0$ . However, one may expect that

$$C_1 \gg C_n(n = 2, 3 \dots),$$

since the values  $Bi'$  and  $Bi$  differ slightly from each other in neighbouring intervals  $\Delta t'_{ik}$  and  $\Delta t_{ik}$ .

Having calculated the integrals, the expression (15) may be given in the form most convenient for quantitative calculations:

$$C_n = \frac{\vartheta_0|_{r=0} \cdot R}{p'_1} \cdot \frac{\frac{\sin(p'_1 - p_n)}{p'_1 - p_n} \frac{\sin(p'_1 + p_n)}{p'_1 + p_n}}{1 - \frac{\sin 2 p_n}{2 p_n}} \quad (16)$$

The calculation of the amplitudes  $A_n = C_n \cdot p_n/R$  for typical regimes of the determination of heat capacity and heat transfer

coefficient at  $Bi = 0.2-0.4$  shows that even at wittingly heightened  $|(Bi - Bi')/Bi| = 10$  per cent, the values  $A_n \approx 0.01 A_1$ , i.e. really all the coefficients of equation (8) are small in comparison with the first one. Taking into account also that the numbers  $m_n$  increase with  $n$ , one may suppose that in this case  $\tau_{reg}, \Delta t_{reg} \rightarrow 0$ .

Thus, when thermal coefficients vary under the conditions of steady-state regular regime at small values of  $Bi$ , then the interval of averaging  $\bar{\Delta t}$  is practically determined only by the interval  $\Delta t_{ik}$  and by the volumetric temperature drop  $\Delta t_R$  (2b).

(5) *Single-valued dependence of the rate  $m$  on the temperature  $\vartheta$*

It is useful to show when substantiating the method of determining the variable coefficients that at the constant temperature of the medium  $t_f = idem$  the rate  $m$  is simply determined by the value  $\vartheta$ , irrespective of the initial  $\vartheta_0$ .

This condition arises logically from the regime regularization concept: the character of the previous thermal regime of a specimen should not influence the value  $m$  after the expiry of the time  $\tau_{reg}$ .

This conclusion is confirmed by experiments. In our work the experiments on cooling the metallic specimens in various media in a wide temperature range  $\vartheta_0 \approx 70^\circ$  were duplicated systematically by experiments on cooling the specimens which had been heated to usual excessive temperature  $\vartheta'_0 \approx 20^\circ$  at the same  $t_f = idem$ . And in the general interval  $\vartheta_0$  the rate  $m$  coincides for both groups of experiments in the range of the usual experimental error.

**3. APPLICATION OF THE METHOD DETERMINING THE VARIABLE COEFFICIENTS**

The method of determining the variable  $\alpha$  was applied in the present work to the systematic investigation of external heat transfer at flowing of a spherical specimen in water and oil [9]. The results obtained were used when calculating the variable heat capacity of specimens of steel (of various trade marks) and nickel.

(1) *Determination of nickel heat capacity in the region of the Curie point*

An electrolytic nickel N2 GOST 849-49 (99-2

per cent Ni) was investigated. A nickel specimen was cooled in the medium (high viscous dead mineral oil) in intersecting temperature intervals at  $t_f = 310$  and  $330^\circ\text{C}$ . The average temperature of a boundary layer  $t_m = \frac{1}{2}(t_f + t_w)$  was the determining one for the coefficient of heat transfer. It is seen from Fig. 2 that experimental curves  $m = f(t_m)$  obtained when treating photographic records of specimen cooling passes through the

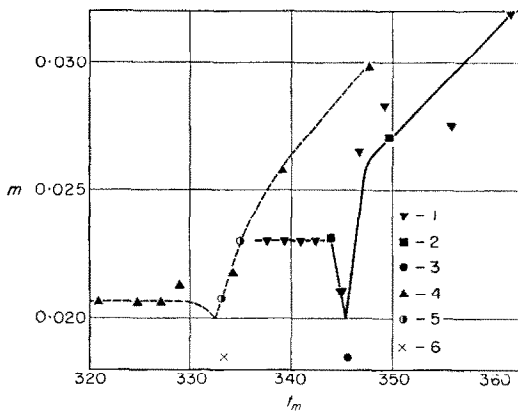


FIG. 2. Results of treating photograms of nickel cooling in the region of the Curie point,  $m$  is in  $\text{s}^{-1}$ .

- 1— $t_f = 330^\circ$ ,  $\Delta\tau_{ik} = 6-8$  s;
- 2— $t_f = 330^\circ$ ,  $\Delta\tau_{ik} = 4$  s;
- 3— $t_f = 330^\circ$ ,  $\Delta\tau_{ik} = 2$  s;
- 4— $t_f = 310^\circ$ ,  $\Delta\tau_{ik} = 6-8$  s;
- 5— $t_f = 310^\circ$ ,  $\Delta\tau_{ik} = 4$  s;
- 6— $t_f = 310^\circ$ ,  $\Delta\tau_{ik} = 2$  s.

minimum corresponding to the heat capacity peak at the Curie point. Both the curves are similar and  $10^\circ$  shifted along the axis  $t_m$  which calls forth their coincidence when plotting in co-ordinates  $m = f(t)$  (where  $t$  is the average temperature of the specimen). Really, the heat capacity values  $c_p(t)$  calculated for both groups of experiments lie on the common curve with highly satisfactory accuracy (Fig. 3). This is indicative of the reliability of the experimental results.

The curve  $c_p(t)$  forms a clear-cut heat capacity peak at the temperature  $t = 362.5^\circ\text{C}$ . The configuration of the peak is typical for heat capacity of a ferromagnetic specimen at the Curie point. The height of the heat capacity peak  $c_{p, \max}$  has a finite value (the transformation of the second kind) and it depends consider-

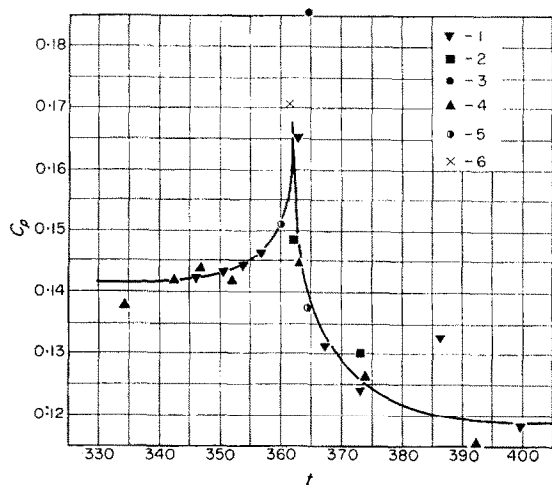


FIG. 3. Heat capacity (cal/g degree) of nickel in the region of the Curie point (by the regular regime method for variable coefficients)

- 1— $t_f = 330^\circ$ ,  $\Delta\tau_{ik} = 6$  s;
- 2— $t_f = 330^\circ$ ,  $\Delta\tau_{ik} = 4$  s;
- 3— $t_f = 330^\circ$ ,  $\Delta\tau_{ik} = 2$  s;
- 4— $t_f = 310^\circ$ ,  $\Delta\tau_{ik} = 6$  s;
- 5— $t_f = 310^\circ$ ,  $\Delta\tau_{ik} = 4$  s;
- 6— $t_f = 310^\circ$ ,  $\Delta\tau_{ik} = 2$  s.

ably on the interval of treating the photograms of  $\Delta\tau_{ik}$ , i.e. on the interval of averaging the thermal constants  $\overline{\Delta t}$ . The contraction of the intervals  $\Delta\tau_{ik}(\Delta t_{ik})$  makes it possible to approach the true heat capacity  $c_{p, \max}$  but, on the other hand, error arises as a result of inaccuracy in measuring adjacent ordinates  $\vartheta$  on the photogram. The optimum value  $\Delta\tau_{ik} = 4$  s at  $t_f = 310^\circ$  or  $\Delta\tau_{ik} = 6$  s at  $t_f = 330^\circ$  was established, which corresponds to the interval of averaging the heat capacity  $\overline{\Delta t} \approx 10^\circ$  both to the height of the peak  $c_{p, \max} = 0.165-0.170$  cal/g degree and to the value of the heat capacity jump at the Curie point  $\Delta c_{p, \max} = 0.045-0.050$  cal/degree. And the error of  $\Delta m_{ik}/m_{ik} = \pm 1.5$  per cent, the total error of  $(\Delta c_p/c_p)_{\max} = \pm 3.5$  per cent.

The curve  $c_p(t)$  obtained coincides practically with that adduced in the literature, but the values  $c_{p, \max}$  and  $\Delta c_{p, \max}$  are higher than in other works ( $c_{p, \max} = 0.155$ ;  $\Delta c_{p, \max} = 0.035$  cal/g degree). To explain this divergence we carried out the analysis of the experimental installations used by Sykes and Wilkinson [13], Klinkhardt [14] and Moser [15] from the point of view of the



value of the temperature interval of averaging the heat capacity  $\overline{\Delta t}$ . In each of these installations the rate of heating the specimen, temperature drop, duration of measuring processes, inertia of measuring instruments and the dissociation of experimental points on the curve  $c_p(t)$  determine the value  $\overline{\Delta t}$ . It turned out that everywhere the value  $\overline{\Delta t} > 10^\circ$ , and it may lead to the understating of  $c_{p, \max}$  in the published works

The control determination of the heat capacity of nickel specimens which were of the same trade mark, N2, was carried out on the installation built in the Leningrad Polytechnical Institute [16]. Particular attention was paid in these experiments to the lowering of  $\Delta t$  (to  $2^\circ$ ). And the increase of  $c_{p, \max}$  compared to the data adduced in the literature was confirmed. The values  $\Delta c_{p, \max} = 0.045$  cal/g degree and Curie temperature coincided with the data of the regular regime method.

The considered question on the correct measurement of the true value of  $c_{p, \max}$  and  $\Delta c_{p, \max}$  has not only a methodological importance, but it is important also for physics of a solid body and, in particular, for the evaluation of the number of magnetons per atom and for more precise definition of the ferromagnetism theory.

### (2) Definition of the carbon steel heat capacity in the region of $A_0$ point

The temperature dependence of heat capacity of some carbon steel specimens in the region of the cementite magnetic transformation ( $A_0$  point) obtained by the regular regime method for the variable coefficients is given in Fig. 4. The interval of averaging the heat capacity is estimated in  $\overline{\Delta t} = 9.5^\circ$ , experimental errors of  $\Delta m_{ik}/m_{ik} = \pm 1.5$  per cent;  $(\Delta c_p/c_p)_{\max} = \pm 3.5$  per cent.

The clear-cut heat capacity maximum is observed in  $A_0$  point. This corroborates the data obtained by Umino [17], the only author who found the heat capacity peak in the  $A_0$  region.

The efficiency of applying the regular regime method in the present problem is apparently conditioned by a photographic record of the continuous cooling curve at rather low values of  $\Delta m_{ik}/m_{ik}$  and  $\overline{\Delta t}$ .

The validity of the results obtained for the

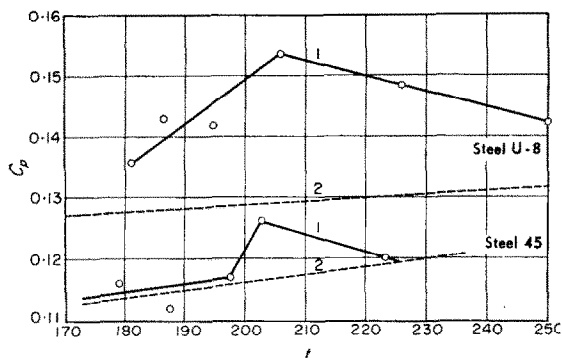


FIG. 4. Heat capacity (cal/g degree) of carbon steel in the region of  $A_0$  point (by the regular regime method for variable coefficients)

1—author's data,  $t_f = 165^\circ\text{C}$ ,  
2—data from References [18–21]

carbon steel is confirmed by the fact that heat capacity of the austenitic and ferrite steel, measured by the same method, increases slightly and monotonously with temperature in the same region forming no maximum.

### REFERENCES

1. G. M. KONDRAT'EV, *Regulyarni Teplovoi Rezhim*, GTTI (1954).
2. E. M. SEMYASHKIN, *Sravnitel'noi opredelenie srednekoeffitsiyentov teplotdachi tsilindrov po metodam stadionarnogo i regul'yarnogo rezhimov*, LITMO Sbornik Statei, Vyp. 8, 27 (1953).
3. G. N. DULNEV and E. M. SEMYASHKIN, *Sravnienie dvukh metodov izmereniya koeffitsienta teplotdachi*, LITMO Sbornik Statei, Vyp. 21, 233 (1957).
4. E. J. NEKHENDZI, *Opredelenie teplovykh konstant metallov metodom regul'yarnogo rezhima*, avtoreferat dissertatsii, Leningrad (1956).
5. *Pervaya mezhvuzovskaya nauchno-tekhnicheskaya konferentsiya po teplovomu regul'yarnomu rezhimu, Kratkoe Soderzhanie Dokl.* LITMO, Leningrad (1958); *Inzh. Fiz. Zh.* 1, 8 (1958).
6. L. I. KUDRYASHOV and A. V. TEMNIKOV, *Obobshchenie teorii teplovogo regul'yarnogo rezhima na sluchai peremennykh teplofizicheskikh karakteristik*, *Inzh. Fiz. Zh.* 1, 10 (1958).
7. L. I. KUDRYASHOV and V. G. ZHEMKOV, *Obobshchenie teorii teplovogo regul'yarnogo rezhima na sluchai peremennykh teplofizicheskikh karakteristik*, *Inzh. Fiz. Zh.* 2, 4 (1959).
8. A. V. LUIKOV, *Teoriya Teploprovodnosti*, GTTI (1952).
9. E. J. NEKHENDZI, *Opredelenie teploemkosti i teploprovodnosti metallov v usloviyakh regul'yarnogo rezhima*, *Zh. Tekh. Fiz.* 24, 8 (1954).

10. C. SYKES, Methods for investigating thermal changes occurring during transformations in a solid solution, *Proc. Roy. Soc. A* **148**, 864, 422 (1935).
11. C. SMITH, Prostoi termicheskii metod kolichestvennogo opredeleniya udelnoi teploemkosti i skrytoi teploty prevrashchenii (referat), *Metallurg.* **7**, 57 (1940).
12. Y. A. TUROVSKII and G. M. BARTENEV, Opredelenie teploemkosti metallov metodom okhlazhdeniya, *Zh. Tekh. Fiz.* **10**, 6, 514 (1940).
13. C. SYKES and H. WILKINSON, The specific heat of nickel from 100° to 600°C, *Proc. Phys. Soc.*, **50**, 281, 834 (1938).
14. H. KLINKHARDT, Messung von wahren spezifischen Wärmen bei hohen Temperaturen durch Heizung mit Glühelektronen, *Annalen Physik*, **84**, 17, 167 (1927).
15. H. MOSER, Messung der wahren spezifischen Wärme von Silber, Nickel,  $\beta$ -Messung, Quarzkristall und Quarzglas zwischen +50 und 700°C nach einer verfeinerten Methode, *Z. Phys.* **37**, 21, 737 (1936).
16. G. M. FEENBERG and Y. A. BERMAN, Opredelenie ictinnoi teploemkosti tverdykh tel v shirokom intervale temperatur, *Zavodskaya Laboratoriya*, **9**, 1057 (1953).
17. S. UMINO, *Sci. Rep. Tohoku Univ.* **16**, 798, 1027 (1927); **23**, 665 (1935).
18. C. SYKES and H. EVANS, Specific heat temperature curves of commercially pure iron and certain plain carbon steels, *J. Iron St. Inst.* **138**, 2, 125 (1938).
19. J. H. AWBERY and A. SNOW, Total heat at various temperatures up to 950°C, *Iron St. Inst., Spec. Rep.*, No. 24 (1939).
20. P. L. GRUZIN, G. V. KURDUMOV and R. I. ENTIN, O prirode "tretego prevrashcheniya" pri otpuske zakalennoi stali, *Metallurg.* **15**, 8 (128), 15 (1940).
21. N. T. GUDTSOV, (Redaktor) Konstrukshionye stali (spravochnik), T. I., *Metallurgizdat M.* (1947).